647510 **BSCmaMJ205x-3C** **Seat No :________**

B.Sc. Semester - 3 (NEP-2020) Examination
OCT/NOV-2025 (AS PER SYLLABUS YEAR JUNE 2024)
MATH: Linear Algebra -1 (Major-5)

Time: 2:00 Hours **Marks:50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 Answer any two questions out of three. (10)

(a) Prove that set of vectors $\{v_1,v_2,v_3,\ldots\ldots\ldots v_n\}$ is linearly dependent iff vector v_k, $2 \le k \le n$, which is linear combination of its Preceeding vectors $v_1,v_2,v_3,\ldots\ldots\ldots v_{k-1}$.

(b) State and prove necessary and sufficient condition for a non-empty subset W of a vector space V to be a subspace of V.

(c) Check whether vector $(1,5,4) \in SPA$. where set $A=\{(0,1,-1),(0,0,2),(1,3,0)\}$.

Que.2 Answer any two questions out of three. (10)

(a) If W_1 and W_2 be two subspaces of a finite dimensional vector space V then prove That $\dim(W_1+W_2) = \dim W_1+\dim W_2-\dim(W_1 \cap W_2)$.

(b) State and prove Extension theorem for basis of vector space.

(c) Prove that the set $A = \{(1,2,1),(2,1,0),(1,-1,2)\}$ forms a basis of R^3. Find co-ordinates of $(1,1,-1)$ with respect to this base.

Que.3 Answer any two questions out of three. (10)

(a) $T: U \rightarrow V$ be a linear transformation. Prove that if $U = \{u_1,u_2,u_3,\ldots\ldots u_n\}$ then $R_T = SP\{T(u_1),T(u_2),\ldots\ldots\ldots\ldots T(u_n)\}$.

(b) Set $\{u_1,u_2,u_3,\ldots\ldots u_n\}$ is basis of a vector space U and $v_1,v_2,v_3,\ldots\ldots\ldots v_n$ be vectors of vector space V then prove that there exist a unique linear transformation $T:U\rightarrow V$ such that $T(u_i)=v_i$, $1 \le i \le n$.

(c) State Rank-Nullity theorem and find N_T, R_T, $n(T)$, $r(T)$ for linear transformation
$T:R^4 \rightarrow R^2$, $T(a,b,c,d)=(a-b+c-d,\ 2a+b+3c+d)$, Where $(a,b,c,d) \in R^4$.

Que.4 Answer any two questions out of three. (10)

(a) State and Prove Triangle inequality for inner product space.

(b) V be inner product space and $u,v \in V$. Prove that $\|u\|\,\|v\| = |u\cdot v|$ iff set$\{u,v\}$is linearly dependent.

(c) Let R^3 have the Euclidean inner product space. Use Gram-Schmidt process to transform the basis $\{(1,0,0),(1,1,1),(1,2,3)\}$ of R^3 in to orthonormal basis.

Que.5 Answer any two questions out of three. (10)

(a) **(i)** Examine $W=\{(a,b,c) \mid a+b+c=0 ,\ a,b,c \in R\}$ For subspace of R^3.
(ii) If set $A= \{(1,-2,5),(2,1,-1),(3,-1,b)\}$ of vector space R^3 is Linearly dependent then find value of b.

(b) Extend set $B= \{1-x+x^2 ,\ 2x-x^2+x^3\}$ of vector space $P_3(R)$ to form a base of $P_3(R)$.

(c) **(i)** Find linear transformation $T: R^2\rightarrow R^2$ Such that $T(0,1)=(3,4)$; $T(3,2)=(5,7)$; $T(3,1)=(2,2)$, if exist.
(ii) if u and v are vectors of real inner product space V such that $\|u\|=\|v\|$ then Prove that $(u+v)\cdot(u-v)=0$.

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